



The comprehensive influence of several major irreversibilities on the performance of an Ericsson heat engine

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Abstract

An irreversible cycle model of Ericsson heat engines including finite-rate heat transfer, regenerative losses and heat leak losses, is established and used to investigate the influence of these irreversibilities on the performance of an Ericsson heat engine, using an ideal gas as the working substance. The power output is adopted as an objective function for optimization. The influence of major irreversibilities on the maximum power output and the corresponding efficiency is analyzed in detail. The inherent relations and the essential differences between the Ericsson heat engine and the Carnot heat engine are expounded. Moreover, it is pointed out that the results obtained here are also suitable for the Stirling heat engine. © 1998 Published by Elsevier Science Ltd. All rights reserved.

Keywords: Ericsson heat engine; Finite-rate heat transfer; Regenerative loss; Heat leak loss; Maximum power output; Maximum efficiency

1. Nomenclature

C	heat capacity of the working substance per mole in the regenerative processes
k_1	thermal conductance between the working substance and the heat source
k_2	thermal conductance between the working substance and the heat sink
n	mole number of the working substance
P	power output
P_1	pressure of the working substance in the isobaric compressive process

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P_2	pressure of the working substance in the isobaric expansive process
P_m	power output at maximum efficiency
$P_{\max}$	maximum power output
Q_1	heat absorbed from the heat source by the working substance in the high-temperature isothermal process
Q_2	heat released to the heat sink by the working substance in the low-temperature isothermal process.
Q_c	heat released to the heat sink per cycle
Q_h	heat released from the heat source per cycle
Q_L	heat leak losses
ΔQ_r	regenerative losses
R	universal gas constant
ΔS	the entropy change of the working substance in the isothermal processes
t	cyclic period
t_1	time of the high-temperature isothermal process
t_2	time of the low-temperature isothermal process
t_3	time of a regenerative process
T_1	temperature of the working substance in the high-temperature isothermal process
T_2	temperature of the working substance in the low-temperature isothermal process
T_c	temperature of the heat sink
T_h	temperature of the heat source
η	efficiency
η_c	efficiency of a reversible Carnot heat engine
η_{cm}	maximum power efficiency of an endoreversible Carnot heat engine without the heat leak losses
$\eta_{em,lr}$	maximum power efficiency of an Ericsson heat engine with the regenerative losses and heat leak losses
$\eta_{em,r}$	maximum power efficiency of an Ericsson heat engine with the regenerative losses
$\eta_{em,l}$	maximum power efficiency of an Ericsson heat engine with the heat leak losses
$\eta_{\max}$	maximum efficiency of an Ericsson heat engine
η_r	efficiency of the regenerator

2. Introduction

In recent years, the use of the Ericsson cycle has been considered in electrothermodynamic power convertors [1], solar-powered heat engines [2], refrigeration applications [3,4], and so on. Many authors have used the reversible and endoreversible models to discuss the performance characteristics of the Ericsson cycle. In this paper, we mainly analyze the influence of several major irreversibilities on the performance of an Ericsson heat engine. The results obtained here will be useful for the further understanding of an Ericsson heat engine and for the selection of the optimal operating conditions.

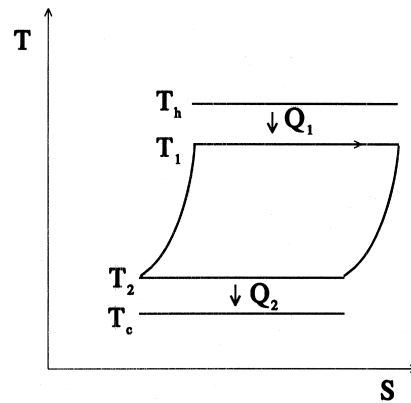


Fig. 1. Schematic diagram of an Ericsson cycle.

3. An irreversible cycle model

It is well known that the working substance of an Ericsson cycle may be a gas, a magnetic material, etc. For different working substances, the Ericsson cycle will have different performance characteristics. When the working substance in the Ericsson cycle is assumed to be an ideal gas, the Ericsson cycle consists of two isothermal and two isobaric processes, as shown in Fig. 1, in which Q_1 and Q_2 are, respectively, the amounts of heat absorbed from the heat source at temperature T_h and released to the heat sink at temperature T_c by the working substance during two isothermal processes and are given by:

$$Q_1 = T_1 \Delta S \quad (1)$$

and

$$Q_2 = T_2 \Delta S, \quad (2)$$

where ΔS is the entropy change of the working substance in the two isothermal processes. It may be expressed by:

$$\Delta S = nR \ln(P_2/P_1) \quad (3)$$

where n is the mole number of the working substance, R is the universal gas constant, P_1 and P_2 are the pressures of the working substance during two isobaric regenerative processes, and $P_1 < P_2$.

For a real Ericsson heat engine, there usually exists thermal resistances between the working substance and the external heat reservoirs at temperatures T_h and T_c . In order to obtain a non-zero power output, the temperatures T_1 and T_2 of the working substance in the two isothermal processes must differ from those of the heat reservoirs so that the two isothermal processes can be finished in a finite time. When the influence of thermal resistances is considered, the performance of the Ericsson cycle depends on the heat transfer law. It is often assumed that heat transfer obeys a Newtonian law [5–7]. Then Q_1 and Q_2 may be written as:

$$Q_1 = k_1(T_h - T_1)t_1 \quad (4)$$

and

$$Q_2 = k_2(T_2 - T_c)t_2, \quad (5)$$

respectively, where k_1 and k_2 are the thermal conductances between the working substance and the heat reservoirs at temperatures T_h and T_c , and t_1 and t_2 are the times of the two isothermal processes at temperatures T_1 and T_2 , respectively.

There also exists the influence of the irreversibility of finite-rate heat transfer in the regenerative processes besides the two isothermal processes, so that the Ericsson cycle doesn't, in general, possess the condition of perfect regeneration. The heat ΔQ_r must be released to the heat sink at temperature T_c in one regenerative process, and absorbed from the heat source at temperature T_h in the other regenerative process, by the working substance. It is assumed reasonably that the regenerative losses per cycle is determined by:

$$\Delta Q_1 = nC(1 - \eta_r)(T_1 - T_2), \quad (6)$$

where C is the heat capacity of the working substance per mole in the regenerative processes, η_r is the regenerative efficiency of the regenerator. When $\eta_r = 1$, the Ericsson cycle possesses the condition of perfect regeneration.

Owing to the influence of the irreversibility of finite-rate heat transfer, the regenerative processes need a non-negligible time compared with the time of the two isothermal processes. In order to calculate the time of the regenerative processes, it is assumed that the temperature of the working substance in the regenerative processes as a function of time is given by [8]

$$dT/dt = \pm u, \quad (7)$$

where u is a proportional constant which is independent of temperature but dependent on the property of the regenerative material, and the positive and negative signs correspond to the isobaric expansive and compressive processes, respectively. From Eq. (7), we obtain the time of the two isobaric processes as:

$$t_3 = t_4 = (T_1 - T_2)/u. \quad (8)$$

It is clearly seen from Eq. (8) that the time of the regenerative processes is assumed to be directly proportional to the temperature difference of the working substance in two isothermal processes. Thus, the cyclic period is given by:

$$t = t_1 + t_2 + 2t_3 = \frac{T_1 \Delta S}{k_1(T_h - T_1)} + \frac{T_2 \Delta S}{k_2(T_2 - T_c)} + \frac{2}{u}(T_1 - T_2). \quad (9)$$

In addition, the heat-leak losses from the heat source at temperature T_h to the heat sink at temperature T_c is directly proportional to the cycle time and given by:

$$Q_L = k_L(T_h - T_c)t, \quad (10)$$

where k_L is the heat leak coefficient between the heat source and the heat sink.

When the three major irreversibilities mentioned above are taken into account, the net heats Q_h released from the heat source and Q_c released to the heat sink per cycle are, respectively,

given by:

$$Q_h = Q_1 + \Delta Q_r + Q_L \quad (11)$$

and

$$Q_c = Q_2 + \Delta Q_r + Q_L. \quad (12)$$

Such a cycle model is useful because it includes not only the influence of thermal resistances between the working substance and the heat reservoirs, but also the regenerative losses of the regenerative processes and the heat leak losses of the heat source. It may be used to derive the maximum power output and discuss other optimal performances of the Ericsson cycle.

4. Maximum power output

From Eqs. (1), (2), (6) and (9)–(12), we obtain the power output

$$P = \frac{Q_h - Q_c}{t} = \frac{Q_1 - Q_2}{t} = \frac{T_1 - T_2}{\frac{T_1}{k_1(T_h - T_1)} + \frac{T_2}{k_2(T_2 - T_c)} + b(T_1 - T_2)}, \quad (13)$$

and the efficiency

$$\eta = \frac{Q_h - Q_c}{Q_h} = \frac{Q_1 - Q_2}{Q_h} = \frac{T_1 - T_2}{T_1 + a(T_1 - T_2) + E \left[\frac{T_1}{k_1(T_h - T_1)} + \frac{T_2}{k_2(T_2 - T_c)} + b(T_1 - T_2) \right]}, \quad (14)$$

where $a = C(1 - \eta_r)/[R \ln(P_2/P_1)]$, $b = 2/[nRu \ln(P_2/P_1)]$ and $E = k_L(T_h - T_c)$.

Using the extremal conditions

$$\frac{\partial P}{\partial T_1} = 0, \quad \frac{\partial P}{\partial T_2} = 0, \quad (15)$$

and Eq. (13), one can find that when the power output attains its maximum, the temperatures T_1 and T_2 of the working substance in the two isothermal processes are determined by:

$$T_1 = T_h \frac{\sqrt{k_1/k_2} + \sqrt{T_c/T_h}}{1 + \sqrt{k_1/k_2}} \quad (16)$$

and

$$T_2 = T_c \frac{1 + \sqrt{k_1/k_2} \sqrt{T_h/T_c}}{1 + \sqrt{k_1/k_2}}. \quad (17)$$

Substituting Eqs. (16) and (17) into Eqs. (13) and (14), we obtain the maximum power output:

$$P_{\max} = \frac{KT_h \eta_{\text{cm}}^2}{1 + F \eta_{\text{cm}}^2}, \quad (18)$$

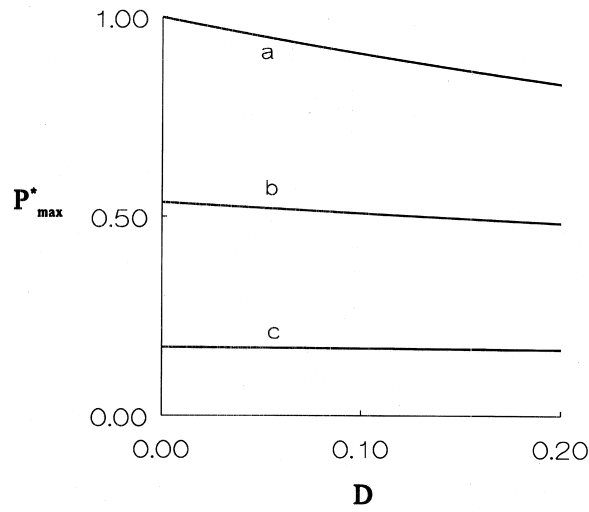


Fig. 2. The dimensionless maximum power output $P_{\max}^* [=P_{\max}/(KT_c)]$ vs the parameter $D (=bKT_c)$. Curves a, b and c correspond to the cases of $T_h/T_c=4, 3$ and 2 , respectively.

with the corresponding efficiency

$$\eta_{\text{em,lr}} = \frac{\eta_{\text{cm}}}{1 + a\eta_{\text{cm}} + E(1 + F\eta_{\text{cm}}^2)/(KT_h\eta_{\text{cm}})}, \quad (19)$$

where $K = k_1k_2/(\sqrt{k_1} + \sqrt{k_2})^2$, $F = bKT_h$ and $\eta_{\text{cm}} = 1 - \sqrt{(T_c/T_h)}$ is the maximum power efficiency of an endoreversible Carnot heat engine [5–7, 9] without the heat-leak losses.

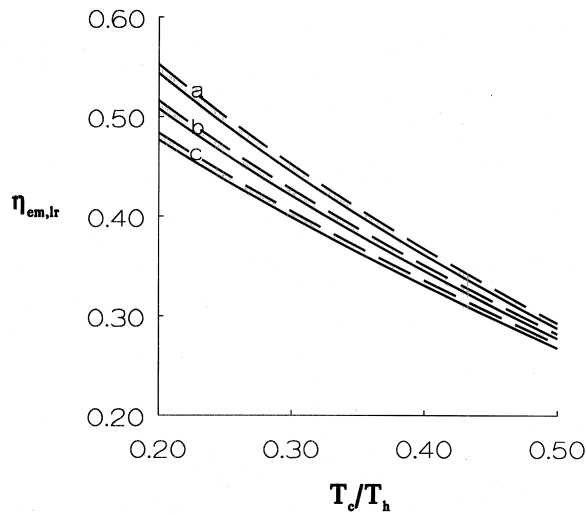


Fig. 3. The maximum power efficiency $\eta_{\text{em,lr}}$ vs the ratio T_c/T_h of the temperatures of two heat reservoirs. Plots are presented for $C/R = 2.5$, $P_2/P_1 = 2.639$, $D = 0.1$ and $k_L/K = 0$ (dashed lines) and 0.01 (solid lines). Curves a, b and c correspond to the cases of $\eta_r = 1, 0.95$ and 0.9 , respectively.

It is seen from Eq. (18) that the maximum power output of the Ericsson heat engine is not affected by the regenerative losses ΔQ_r and the heat leak losses Q_L although it is dependent on the time of the regenerative processes. We can prove that when the temperature of the working substance in the adiabatic processes of the Carnot heat engine varies with the relation given by Eq. (7), the expression of the maximum power output of an endoreversible Carnot heat engine with the heat leak losses determined by Eq. (10) is also given by Eq. (18). The influence of b on the maximum power output may be clearly seen from Fig. 2, where $D = bKT_c$ and $P_{\max}^* = P_{\max}/(KT_c)$ is the dimensionless maximum power output of a heat engine. Obviously, the larger the value of T_h/T_c is, the larger the influence of b on the maximum power output is.

Eq. (19) shows clearly that when the Ericsson heat engine operates in the maximum power output, its efficiency is always smaller than η_{cm} , because it is affected by not only the regenerative losses but also the heat leak losses. The curves in Fig. 3 show clearly that the effect of the regenerative losses and heat leak losses on the maximum power efficiency of the Ericsson heat engine is obvious.

When $\eta_r = 1$, the Ericsson heat engine possesses the condition of perfect regeneration although the time of the regenerative processes is considered. The maximum power efficiency of the Ericsson heat engine is given by:

$$\eta_{em,1} = \frac{\eta_{cm}}{1 + E(1 + F\eta_{cm}^2)/(KT_h\eta_{cm})}. \quad (20)$$

In this case, not only the maximum power output but also the maximum power efficiency have the same expressions for both the Ericsson heat engine and the Carnot heat engine operating in the same temperature range and having the same heat-leak losses described by Eq. (10). It is thus clear that the performance characteristics of an endoreversible Carnot heat engine with the heat leak losses may be directly discussed by using Eqs. (18) and (20).

When $k_L = 0$, the maximum power efficiency of the Ericsson heat engine

$$\eta_{em,r} = \frac{\eta_{cm}}{1 + a\eta_{cm}} \quad (21)$$

is different from that of the Carnot heat engine without the heat leak losses.

Incidentally, the special case $\eta_r = 1$ and $k_L = 0$ has been discussed in ref. [8] and other papers.

5. The P – η curves

To understand the optimal performance of an Ericsson heat engine all-sidedly, we should analyze further other operating states besides the state of the maximum power output. From Eqs. (13) and (14), we find that the power output and the efficiency may be, respectively, expressed as:

$$P = \frac{KT_h(1-x)(x - T_c/T_h)}{x + F(1-x)(x - T_c/T_h)} \quad (22)$$

and

$$\eta = \frac{1}{\frac{1}{1-x} + a + E \frac{x + F(1-x)(x - T_c/T_h)}{KT_h(1-x)(x - T_c/T_h)}}, \quad (23)$$

where $x = T_2/T_1$.

It is seen from Eq. (23) that when the heat-leak loss is not considered, η is a monotonous function of x . When $x = T_c/T_h$, $P = 0$ and η attains its maximum, i.e.

$$\eta_{\max} = \frac{1}{\frac{T_h}{T_h - T_c} + a}. \quad (24)$$

When $k_L > 0$, η is not a monotonous function of x . We can determine from Eq. (23) that the maximum efficiency is given by:

$$\eta_{\max} = \frac{1}{a + gF + \frac{(1+g)^2}{g(\sqrt{1+K/k_L} - \sqrt{T_c/T_h})^2}}, \quad (25)$$

with the corresponding power output

$$P_m = \frac{KT_h}{F + \frac{(1+g)(g\sqrt{1+K/k_L} + \sqrt{T_c/T_h})}{g^2\sqrt{1+K/k_L}(\sqrt{1+K/k_L} - \sqrt{T_c/T_h})^2}}, \quad (26)$$

where $g = E/(KT_h)$.

Eliminating x from Eqs. (22) and (23), we obtain the P – η curves which describe the general performance characteristics of an Ericsson heat engine, as shown in Fig. 4, where $P^* = P/(KT_c)$ is the dimensionless power output and η_c is the efficiency of a reversible Carnot cycle. The curves in Fig. 4 show clearly that the Ericsson heat engine should be operated in between the maximum power efficiency and the maximum efficiency.

6. Discussion

Obviously, the performance of an Ericsson cycle is directly dependent on the time as well as the form of regenerative processes. When the time of the regenerative processes is directly proportional to that of the two isothermal processes, i.e.

$$2t_3 = \gamma(t_1 + t_2), \quad (27)$$

we can prove that the maximum power output of the Ericsson heat engine

$$R_{\max} = \frac{KT_h\eta_{cm}^2}{1 + \gamma} \quad (28)$$

is also identical with that of the endoreversible Carnot heat engine [5], in which the time of the

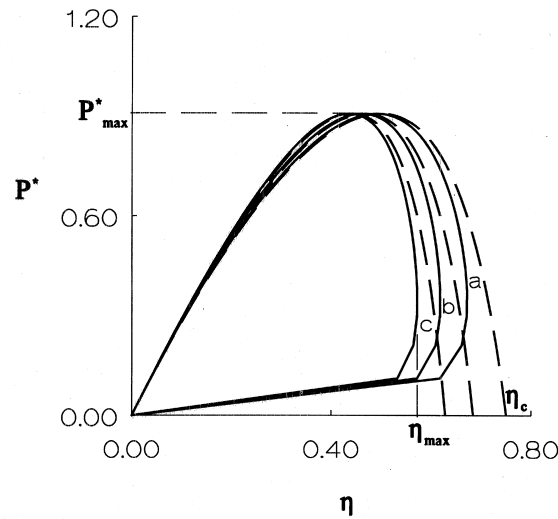


Fig. 4. The dimensionless power output $P^* [= P/(KT_c)]$ vs the efficiency η . The values of C/R , P_2/P_1 , D , k_L/K and η_r are the same as those used in Fig. 3, and $T_h/T_c = 4$.

adiabatic processes is $\gamma(t_1 + t_2)$, and the corresponding efficiency is given by:

$$\eta_{cm} = \frac{\eta_{cm}}{1 + a\eta_{cm} + (1 + \gamma)E/(KT_h\eta_{cm})}, \quad (29)$$

where γ is a proportional constant. Using the similar steps mentioned above, the reader can obtain other significant results.

Although the Ericsson cycle is different from the Stirling cycle, which consists of two isothermal and two constant-volume processes, it is very important to note the fact that all the results in this paper are also true for the Stirling heat engine so long as P_2/P_1 is replaced by V_2/V_1 , where V_2 and V_1 ($V_2 > V_1$) are, respectively, the volumes of the working substance in the two constant-volume processes.

7. Conclusions

The regeneration is one of the important characteristics in the Ericsson and Stirling cycles. In particular, when finite-rate heat transfer is considered, the regenerative processes in the Ericsson and Stirling cycles are affected by thermal resistances, so that the regenerative losses are always unavoidable. Although the regenerative losses don't affect the maximum power output of the Ericsson and Stirling heat engines, it always reduces the maximum power efficiency. This is an important characteristic of the Ericsson and Stirling cycles affected by the irreversibility of finite-rate heat transfer. The results obtained above show clearly that in further investigations of the Ericsson and Stirling heat engines, it would be impossible to obtain other conclusions than for the Carnot cycle if the regenerative losses were not taken into account.

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